Type-II :-

Eequation's of the form

$$f(z, P, q) = 0$$

not containing independent variable x, y.

Let us assume $z = f(x + ay)$ as a total solutions.

Put $z = f(X)$ Where $X = x + ay$

Here $p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial X} \cdot \frac{\partial X}{\partial x} = \frac{\partial z}{\partial X} \cdot 1 = \frac{\partial z}{\partial X}$ &

$$q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial X} \cdot \frac{\partial X}{\partial y} = \frac{\partial z}{\partial X} \cdot a = a \cdot \frac{\partial z}{\partial X}$$

Now, this form $f(z, P, q) = 0$ becomes

$$f\left(z, \frac{\partial z}{\partial X}, a\frac{\partial z}{\partial X}\right) = 0$$ which

is an ordinary differential equation.

(Q) Example, $z = Pq$

Sol^n:→ This is in the form $f(z, P, q) = 0$

Putting $p = \frac{dz}{dX}$ & $q = a\frac{dz}{dX}$

Where $X = x + ay$, weget

$$z = \left(\frac{dz}{dX}\right)\left(a \cdot \frac{dz}{dX}\right) = a\left(\frac{dz}{dX}\right)^2$$

$$\therefore \frac{\sqrt{z}}{\sqrt{a}} = \frac{dz}{dX}$$

$$\therefore dX = \sqrt{a}\,\frac{dz}{\sqrt{z}}$$

integrating $X = 2\sqrt{a}\cdot\sqrt{z} + c$

or, $x + ay = 2\sqrt{z \cdot a} + c$

which is the required solⁿ.

N.B :→ Procedure will be clear by Charpit's methed.

(Q2) Solve $p^2 = z^2(1-pq)$

Solⁿ :→ It is of the form $f(z,p,q) = 0$

Put $p = \frac{dz}{dX}$, $q = a\frac{dz}{dX}$ & $X = x + ay$, we get

$$\therefore \left(\frac{dz}{dX}\right)^2 = z^2\left(1 - \frac{dz}{dX}\cdot a\frac{dz}{dX}\right)$$

$$\therefore \left(\frac{dz}{dX}\right)^2 = z^2 - az^2\left(\frac{dz}{dX}\right)^2$$

$$\therefore \left(\frac{dz}{dX}\right)^2 = \frac{z^2}{1+az^2}$$

$$\therefore \frac{dz}{dX} = \frac{z}{\sqrt{1+az^2}}$$

$$\therefore \frac{\sqrt{1+az^2}}{z}\, dz = dx = \left(\frac{1+az^2}{z\sqrt{1+az^2}}\right)\cancel{dz}$$

$$\therefore dx = \frac{dz}{z\sqrt{1+az^2}} + \frac{az}{\sqrt{1+az^2}}\, dz$$

$$\therefore X + c = \int \frac{dz}{z\sqrt{1+az^2}} + \sqrt{1+az^2}$$

$$\therefore x + ay + c = \int \frac{dz}{z\sqrt{1+az^2}} + \sqrt{1+az^2},$$

is the complet sol^n.

(83) $9(p^2 z + q^2) = 4$

Sol^n:→ It is of the form $f(z,p,q) = 0$

Put $p = \frac{dz}{dX}$, & $q = a\cdot\frac{dz}{dX}$ & $X = x+ay$

Eeq^n is, $9\left[\left(\frac{dz}{dX}\right)^2 z + a^2\left(\frac{dz}{dX}\right)^2\right] = 4$

$$\therefore \frac{dz}{dX}\sqrt{z^2+a^2} = \frac{2}{3}$$

$$\therefore dX = \frac{2}{3}\sqrt{z+a^2}\, dz.$$

integrating

$$X = \frac{3}{2}\,\frac{(z+a^2)^{3/2}}{\frac{3}{2}} + c$$

$$\therefore X = (z+a^2)^{3/2} + c$$

$x + ay = (z+a^2)^{3/2} + c$ is complete sol^n.

(Q4) Solve $z^2(p^2+q^2+1) = c^2$

Sol^n:→ It is of the form $f(z, p, q) = 0$

Put $p = \frac{dz}{dX}$, $q = a\cdot\frac{dz}{dX}$ & $X = x + ay$

$\therefore z^2\left[\left(\frac{dz}{dX}\right)^2 + a^2\left(\frac{dz}{dX}\right)^2 + 1\right] = c^2$

or, $\left(\frac{dz}{dX}\right)^2(1+a^2) = \frac{c^2-z^2}{z^2}$

$\therefore \frac{dz}{dX}\sqrt{1+a^2} = \frac{\sqrt{c^2-z^2}}{z}$

$\therefore \frac{z\cdot dz}{\sqrt{c^2-z^2}} = \frac{1}{\sqrt{1+a^2}}\,dX$

integrating

$-\sqrt{c^2-z^2}\,\sqrt{1+a^2} = X + c = x + ay + c$

$\therefore x + ay + \sqrt{(1+a^2)(c^2-z^2)} + c = 0$ is the complete sol^n.